B_c meson and the light-heavy quarkonium spectrum

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Abstract

We compute the $c\bar{b}$ spectrum from a first principle Salpeter equation obtained in a preceding paper. For comparison we report also the heavy-light quarkonium spectrum and the hyperfine separations previously presented only in a graphical form. Notice that all results are parameter free.

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The spectrum and properties of the $c\bar{b}$ systems have been calculated various times in the past in the framework of the heavy quarkonium theory [1]. However the recent experimental observation of the B_c^+ meson [2] has arisen new theoretical interest on the problem [3–5]. The mentioned spectrum has been considered again either from the potential and the lattice simulation point of view. A particular interesting quantity should be the hyperfine splitting that as for the $c\bar{c}$ case seams to be sensible to relativistic and subleading corrections in $\alpha_{\rm s}$.

For the above reasons it seems to us worthwhile to present in this paper a calculation of the $c\bar{b}$ spectrum based on an effective mass operator with full relativistic kinematics which we have obtained in previous works and applied with a certain success to a fit of the entire quarkonium spectrum, heavy-heavy, light-light and light-heavy cases with the exception however of the $c\bar{b}$ case [6,7]. For comparison and completeness we report also numerical results for the light-heavy spectrum which we have given previously only in a graphical form.

The mass operator was obtained by a three dimensional reduction of the $q\bar{q}$ Bethe-Salpeter equation introduced in [8]. It has the quadratic form $M^2 = M_0^2 + U$, with a kinetic part $M_0 = w_1 + w_2 = \sqrt{m_1^2 + \mathbf{k}^2} + \sqrt{m_2^2 + \mathbf{k}^2}$ and a "potential" that in terms of the istantaneous approximation of the B-S kernel is given by

$$\langle \mathbf{k}|U|\mathbf{k}'\rangle = \frac{1}{(2\pi)^3}\sqrt{\frac{w_1+w_2}{2\,w_1 w_2}}\;\hat{I}_{\rm inst}(\mathbf{k},\mathbf{k}')\;\sqrt{\frac{w_1'+w_2'}{2\,w_1' w_2'}}, \tag{1}$$

$\mathbf{k}$ denoting the momentum of the quark in the centre of mass frame, and $i = 1, 2$ the quark and the antiquark.

The B-S equation was derived from QCD first principles, taking advantage of the Feynman-Schwinger path integral representation for the "second order" quark propagator in an external field [1]. The only assumption used consisted in writing the logarithm of the Wilson loop correlator $W = \frac{1}{3}\langle \mathrm{Tr}\, P \exp(\oint dx^\mu A_\mu)\rangle$, as the sum of its perturbative expression and an area term

$$i \ln W = i(\ln W)_{\mathrm{pert}} + \sigma S_{\mathrm{min}}, \tag{2}$$

σ denoting the string tension.

An explicit expression for U is given in Ref. [7]. The perturbative part of such quantity was evaluated at the lowest order in α_{s}. However for α_{s} we have used the standard running expression

$$\alpha_{\mathrm{s}}(\mathbf{Q}) = \frac{4\pi}{(11 - \frac{2}{3}N_{\mathrm{f}}) \ln \frac{\mathbf{Q}^2}{\Lambda^2}} \tag{3}$$

(with $N_{\mathrm{f}} = 4$ and $\Lambda = 200$ MeV) cut at a maximum value $\alpha_{\mathrm{s}}(0)$, to treat properly the infrared region [9]. This amount to include important perturbative subleading contributions.

Notice that, contrary to all the usual potential models, we have given the light quark current and not component masses in our treatment. Component masses of the usual order of magnitude can be recovered at a successive step as effective values in a semirelativistic reformulation [6]. Actually we have fixed such masses on typical values, $m_u = m_d = 10$ MeV, $m_s = 200$ MeV, which are not adjusted in the fit (the results depend essentially on $\langle k \rangle$ and are very little affected by the precise value of the light quark masses). The other parameters of the theory are assumed as: $m_c = 1.394$ GeV, $m_b = 4.763$ GeV, $\sigma = 0.2$ GeV^2, $\alpha_{\mathrm{s}}(0) = 0.35$. The first two are chosen in order to reproduce correctly the J/Ψ and the $\Upsilon(1S)$ masses, the string tension to give the correct slope for the Regge ρ trajectory, $\alpha_{\mathrm{s}}(0) = 0.35$ to give the right $J/\Psi - \eta_c$ splitting. Notice that, consequently, the results reported in this paper are completely parameter free, with the exception of the $c\bar{c}(1S)$ hyperfine splitting.

We have used in our calculations also the more conventional "linear mass" operator (or center of mass relativistic Hamiltonian) $M = M_0 + V$ (where V is defined by $U = M_0 V + V M_0 + V^2$) which makes easier a comparison with the usual phenomenological models. If we neglect the V^2 term, V is obtained from Eq.(1) simply by the kinematical replacement

$$\sqrt{\frac{w_1 + w_2}{2\, w_1 w_2}} \sqrt{\frac{w_1' + w_2'}{2\, w_1' w_2'}} \to \frac{1}{4\sqrt{w_1 w_2 w_1' w_2'}}. \tag{4}$$

This is the form we have used in Ref. [6] (for some state however $\langle V^2 \rangle$ is not negligible). In the calculations based on this linear formalism we have used the same values for the light quark masses as before, a fixed coupling constant $\alpha_{\mathrm{s}} = 0.363$ and taken $m_c = 1.40$ GeV, $m_b = 4.81$ GeV and $\sigma = 0.175$ GeV^2.

Details on the numerical treatment of the eigenvalue equation are given in [6] and [7].

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[1] Second order propagator in the sense that it is defined by a second order differential equation; the quadratic form of the mass operator derives essentially from this fact.

In table I we have reported the $c\bar{b}$ spectrum as obtained by the quadratic and the linear formalism, together with the values presented in Refs. [4] and [5]. The observed mass $M(B_c) = 6.40 \pm 0.39 \pm 0.13$ GeV has to be referred to the $1\,^1\mathrm{S}_0$ state. For such state all calculations give very close results and reproduce equally well the experimental value within the errors. Larger discrepancies among the various methods occur for the excited states.

In table II we have reported the spectrum for light-heavy mesons obtained by our formalism in numerical form. We have considered the hyperfine structure but omitted the fine one. We have also reported the quantity Δ_{avg} defined as the average of the deviations of the theoretical values from the experimental data diminished by the experimental errors. Obviously Δ_{avg} provides a measure of the accuracy in reproducing the data and give an idea of the precision one can expect in the $c\bar{b}$ case.

In table III, finally, we have reported the hyperfine splitting for the $1S$ and $2S$ states as obtained in the quadratic formalism and the Δ_{avg} quantity even for the channels for which we do not reproduce the results in full here.

Notice the strong discrepancies with the data in the hyperfine splittings of the $1S$ light-light cases. This is obviously due to the chiral symmetry breaking problem and the related inadequacy of replacing the quark full propagator in the B-S equation with the free form, as implied in the three-dimensional reduction. For the rest, the agreement is good for the states involving light and c quarks, while the theoretical value tends to be too large for states involving b quarks.

For comparison we can mention that in the linear formalism the hyperfine splitting turns out less good, being e.g. 97 MeV for $c\bar{c}(1S)$, 111 MeV for $u\bar{c}(1S)$, 108 MeV for $c\bar{s}(1S)$. Likely such difference has to be ascribed to relativistic and α_{s} subleading effects, taken into account in the quadratic formalism via Eq.(3).

In conclusion let us mention explicitly that Δ_{avg} as reported in table III do not include the states $c\bar{c}(4S)$ and $b\bar{b}(6S)$, (which are largely above threshold) and the $1\,^1\mathrm{S}_0$ and $1\,^1\mathrm{P}_1$ light-light states for the reasons recalled above.

TABLES

TABLE I. $b\bar{c}$ quarkonium systems. Experimental B_c mass equal to $6.40 \pm 0.39 \pm 0.13$ GeV.

States	quadratic formalism (GeV)	linear formalism (GeV)	Fulcher (GeV)	Lattice (GeV)
$1\,^1S_0$	6.258	6.293	6.286	6.280 ± 0.200
$1\,^3S_1$	6.334	6.355	6.341	6.321 ± 0.200
$2\,^1S_0$	6.841	6.848	6.882	6.960 ± 0.080
$2\,^3S_1$	6.883	6.881	6.914	6.990 ± 0.080
$3\,^1S_0$	7.222	7.221		
$3\,^3S_1$	7.254	7.245		
1 P	6.772	6.762	6.754	6.764 ± 0.030
2 P	7.154	7.138		
1 D	7.043	7.025	7.028	
2 D	7.367	7.346		

TABLE II. Light-Heavy quarkonium systems.

States		experimental values (MeV)	linear formalism (MeV)	quadratic formalism (MeV)
$u\bar{c}$				
$1\,^1\mathrm{S}_0$	$D^{\pm}$	1869.3 ± 0.5	1890	1875
	D^0	1864.5 ± 0.5		
$1\,^3\mathrm{S}_1$	$D^*(2010)^{\pm}$	2010.0 ± 0.5	2001	2020
	$D^*(2007)^0$	2006.7 ± 0.5		
$2\,^1\mathrm{S}_0$	D'	2580	2556	2525
$2\,^3\mathrm{S}_1$	$D^{*\prime}$	2637 ± 8	2615	2606
$1\,\mathrm{P}$	$D_2^*(2460)^{\pm}$	2459 ± 4	2442	2475
	$D_2^*(2460)^0$	2458.9 ± 2.0		
	$D_1(2420)^{\pm}$	2427 ± 5		
	$D_1(2420)^0$	2422.2 ± 1.8		
Δ_{avg}			12	21
$u\bar{b}$				
$1\,^1\mathrm{S}_0$	$B^{\pm}$	5278.9 ± 1.8	5282	5273
	B^0	5279.2 ± 1.8		
$1\,^3\mathrm{S}_1$	B^*	5324.8 ± 1.8	5341	5339
$2\,^1\mathrm{S}_0$			5878	5893
$2\,^3\mathrm{S}_1$	$B^{*\prime}$	5906 ± 14	5916	5933
$1\,\mathrm{P}$		5825 ± 14	5777	5792
Δ_{avg}			34	19
$s\bar{c}$				
$1\,^1\mathrm{S}_0$	$D_s^{\pm}$	1968.5 ± 0.6	1999	1982
$1\,^3\mathrm{S}_1$	$D_s^{*\pm}$	2112.4 ± 0.7	2107	2120
$2\,^1\mathrm{S}_0$			2667	2617
$2\,^3\mathrm{S}_1$			2729	2698
$1\,\mathrm{P}$	$D_{sJ}(2573)^{\pm}$	2573.5 ± 1.7	2528	2548
	$D_{s1}(2536)^{\pm}$	2535.35 ± 0.34		
Δ_{avg}			21	9
$s\bar{b}$				
$1\,^1\mathrm{S}_0$	B_s^0	5369.3 ± 2.0	5373	5364
$1\,^3\mathrm{S}_1$	B_s^*	5416.3 ± 3.3	5433	5429
$2\,^1\mathrm{S}_0$			5974	5985
$2\,^1\mathrm{S}_0$			6014	6024
$1\,\mathrm{P}$	$B_{sJ}^*(5850)$	5853 ± 15	5848	5859
Δ_{avg}			5	4

TABLE III. Hyperfine splitting (MeV)

	$u\bar{c}$	$s\bar{c}$	$u\bar{b}$	$s\bar{b}$	$c\bar{c}$	$c\bar{b}$	$b\bar{b}$	$u\bar{u}$	$u\bar{s}$	$s\bar{s}$
1 S	145	138	66	65	115	77	86	349	298	259
Exp	141(1)	144(1)	46(3)	47(4)	117(2)	-	-	630.5(0.6)	393.92(0.24)	335.3(0.1)
2 S	81	81	40	39	67	42	35	135	130	127
Exp	57	-	-	-	92(5)	-	-	165(103)	-	-
Δ_{avg}	21	19	9	4	20	-	10	19	48	18

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